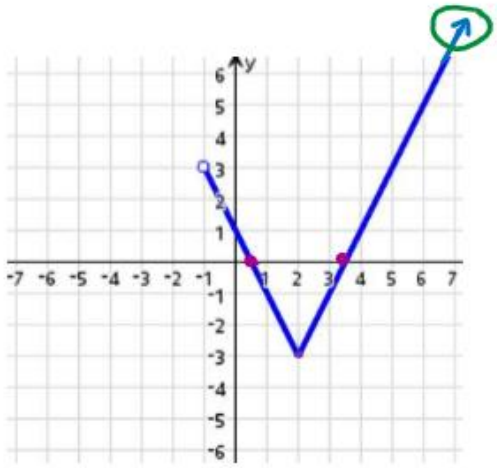
Characteristics of Functions



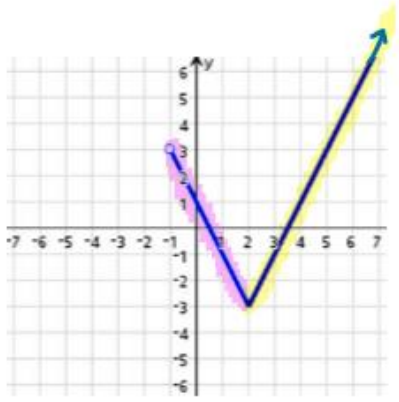
Domain: $(-1, \infty)$ $-1 < x$

Range: $[-3, \infty)$ $-3 \leq y$

Zero(s): .5 , 3.5

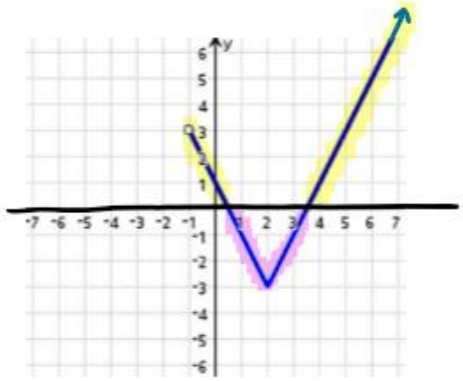
End Behavior: As $x \to -\infty, f(x) \to$ DNE

As $x \to +\infty, f(x) \to \infty$



Increasing Interval(s): $(2, \infty)$

Decreasing Interval(s): $(-1, 2)$



Positive Interval(s): $(-1, .5) \cup (3.5, \infty)$

Negative Interval(s): $(.5, 3.5)$

a and k affect y
b and h affect x

b and h are liars
b - reciprocal
h - opposite

Transformation Equation

$$g(x) = a \cdot f\left(\frac{1}{b}(x-h)\right) + k$$

Scale Change

$|a| > 1$ vertical stretch

$0 < |a| < 1$ vertical shrink

$|b| > 1$ horizontal stretch

$0 < |b| < 1$ horizontal shrink

	Negative < 0	Positive > 0
a	reflect over x-axis	
b	reflect over y-axis	
h	left	right
k	down	up

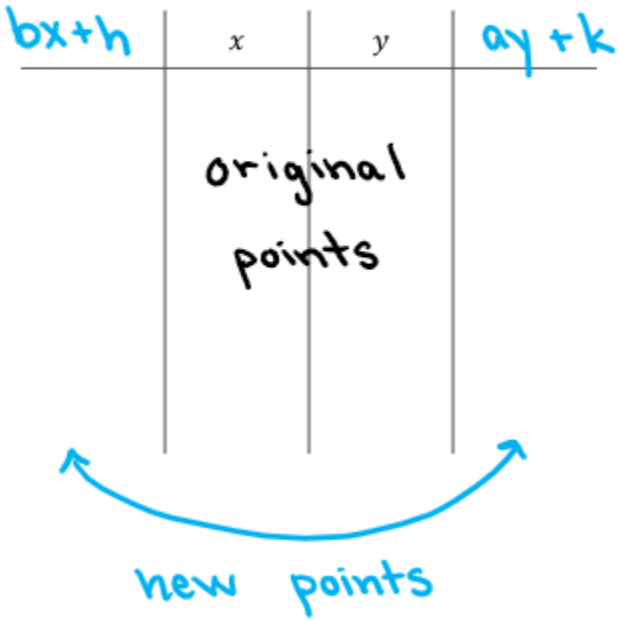
a = 1 no change

b = 1 no change

h = 0 no change

k = 0 no change

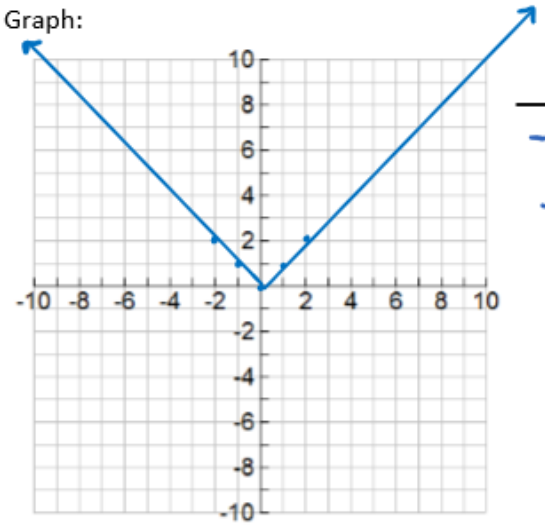
$$g(x) = a \cdot f\left(\frac{1}{b}(x-h)\right) + k$$



Parent Function Name: Absolute Value

Equation: $y = |x|$

Graph:



x	f(x)
-2	2
-1	1
0	0
1	1
2	2

Transformation Equation: $y = a\left|\frac{1}{b}(x-h)\right| + k$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

End Behavior: As $x \to -\infty, f(x) \to \infty$

As $x \to +\infty, f(x) \to \infty$

x-intercept(s): (0, 0)

y-intercept: (0, 0)

Vertex ~~or Critical Point~~: (0, 0)

Axis of Symmetry: $x = 0$

Asymptotes: none

Needed to Write Equation Let $b = 1$

vertex (h, k)

point (x, y)

Solve for a $\quad y = a|x-h| + k$

Additional Notes:

Solving Abs. Value Equations

1. Isolate
2. Separate – drop
   – drop and flip
3. Solve
4. Check

Solving Abs. Value Inequality

1. Isolate
2. Separate w/ joining Word
   less than**and** great**or**
3. Solve
4. Or → graph
   And → combine into one statement

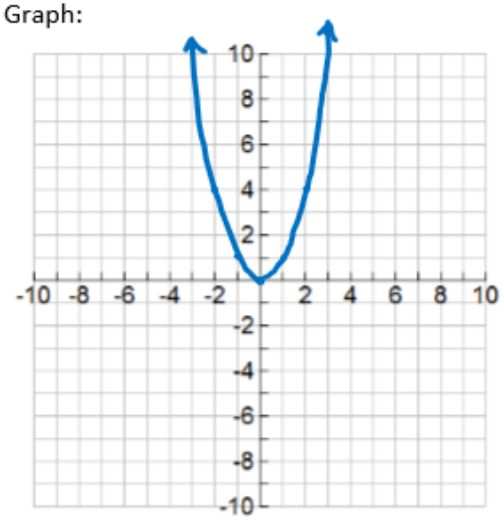
Word Problem

$|x - \text{actual}| \leq \text{tolerance}$

Parent Function Name: Quadratic

Equation: $y = x^2$

Graph:



x	f(x)
-2	4
-1	1
0	0
1	1
2	4

Transformation Equation: $y = a\left(\frac{1}{b}(x-h)\right)^2 + k$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

End Behavior: As $x \to -\infty, f(x) \to \infty$

As $x \to +\infty, f(x) \to \infty$

x-intercept(s): (0, 0)

y-intercept: (0, 0)

Vertex ~~or Critical Point~~: (0, 0)

Axis of Symmetry: $x = 0$

Asymptotes: none

Needed to Write Equation

vertex form	intercept form	standard form
$y = a(x-h)^2 + k$	$y = a(x-p)(x-q)$	$y = ax^2 + bx + c$
vertex (h, k)	x-int $(p, 0)$ $(q, 0)$	regression on calculator
point (x, y)	point (x, y)	
solve for a	solve for a	

Additional Notes:

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Imaginary Numbers

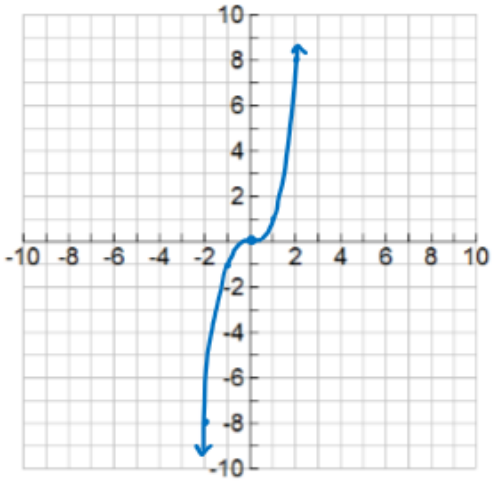
$i = \sqrt{-1}$

$i^2 = -1$

Parent Function Name: Cubic

Equation: $y = x^3$

Graph:



x	f(x)
-2	-8
-1	-1
0	0
1	1
2	8

Transformation Equation: $y = a\left(\frac{1}{b}(x-h)\right)^3 + k$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

End Behavior: As $x \to -\infty, f(x) \to -\infty$

As $x \to +\infty, f(x) \to \infty$

x-intercept(s): (0, 0)

y-intercept: (0, 0)

~~Vertex or~~ Critical Point: (0,0) ← slide point

almost horizontal

Axis of Symmetry: none

Asymptotes: none

Needed to Write Equation Let b = 1

slide point (h,k)

point (x,y)

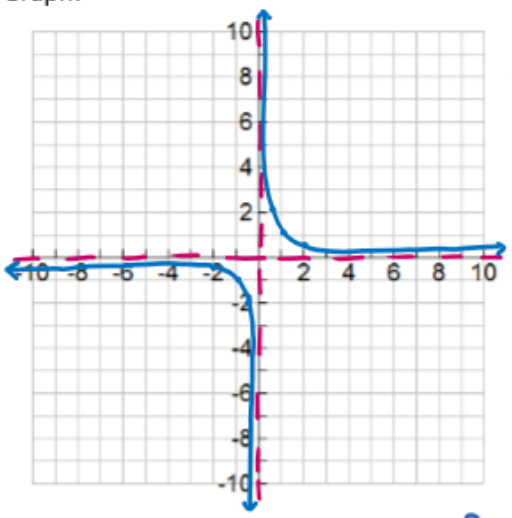
Solve for a

Additional Notes:

Parent Function Name: Rational

Equation: $y = \frac{1}{x}$

Graph:



x	f(x)
-2	-½
-1	-1
-½	-2
0	DNE
½	2
1	1
2	½

Transformation Equation: $y = \frac{a}{\frac{1}{b}(x-h)} + k$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

End Behavior: As $x \to -\infty, f(x) \to$ 0

As $x \to +\infty, f(x) \to$ 0

x-intercept(s): none

y-intercept: none

~~Vertex or~~ Critical Point: intersection of asymptotes

Axis of Symmetry: $y = \pm x$

Asymptotes: VA $x = 0$, HA $y = 0$

Needed to Write Equation Let $b = 1$

VA $x = h$

HA $y = k$

point (x, y)

Solve for a

Additional Notes:

Vertical asymptote

set denominator = 0, solve for x

horizontal asymptote

BOBO - bigger on bottom, $y = 0$

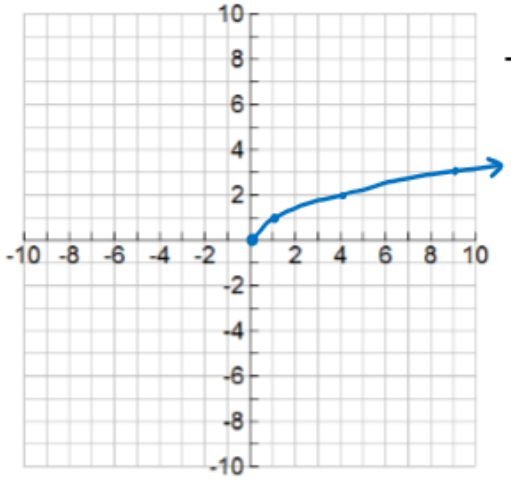
BOTN - bigger on top, none

EATS DC - exponents are the same, divide (leading) coefficients

Parent Function Name: Square Root

Equation: $y = \sqrt{x}$

Graph:



x	f(x)
-1	DNE
0	0
1	1
4	2
9	3

Transformation Equation: $y = a\sqrt{\frac{1}{b}(x-h)} + k$

Domain: $[0, \infty)$

Range: $[0, \infty)$

End Behavior: As $x \to -\infty, f(x) \to$ DNE

As $x \to +\infty, f(x) \to \infty$

x-intercept(s): (0, 0)

y-intercept: (0, 0)

~~Vertex or~~ Critical Point: (0, 0)

Axis of Symmetry: none

Asymptotes: none

Needed to Write Equation

Let $a = 1$ OR $b = 1$

Starting point (h, k)

point (x, y)

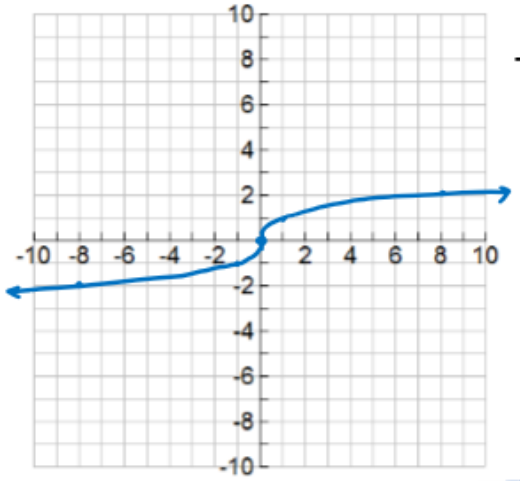
Solve for $\frac{1}{b}$ OR a

Additional Notes:

Parent Function Name: Cube Root

Equation: $y = \sqrt[3]{x}$

Graph:



x	f(x)
-8	-2
-1	-1
0	0
1	1
8	2

Transformation Equation: $y = a\sqrt[3]{\frac{1}{b}(x-h)} + k$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

End Behavior: As $x \to -\infty, f(x) \to -\infty$

As $x \to +\infty, f(x) \to \infty$

x-intercept(s): $(0, 0)$

y-intercept: $(0, 0)$

~~Vertex or~~ Critical Point: $(0,0)$ ← slide point almost vertical

Axis of Symmetry: none

Asymptotes: none

Needed to Write Equation

Let $a = 1$ OR $b = 1$

Slide point (h, k)

point (x, y)

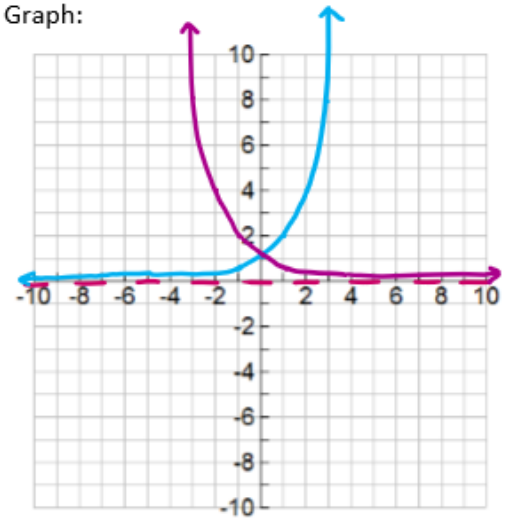
Solve for $\frac{1}{b}$ OR a

Additional Notes:

Parent Function Name: Exponential

Equation: $y = B^x$

Graph:



x	f(x)	Ex: $y = 2^x$	Ex: $y = (\frac{1}{2})^x$
-1	$\frac{1}{B}$	$\frac{1}{2}$	2
0	1	1	1
1	B	2	$\frac{1}{2}$

Transformation Equation: $y = a(B)^{\frac{1}{b}(x-h)} + k$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

End Behavior: (growth, decay)

As $x \to -\infty, f(x) \to$ 0 , ∞

As $x \to +\infty, f(x) \to$ ∞ , 0

x-intercept(s): none

y-intercept: (0, 1)

~~Vertex or~~ Critical Point: (0, 1)

Axis of Symmetry: none

Asymptotes: $y = 0$

Needed to Write Equation $y = a(B)^x + k$

asymptote $y = k$

two points (x_1, y_1) right and (x_2, y_2) left

Solve for B, then a $y = a(B)^{x-h} + k$

Additional Notes:

Growth / Decay

$y = a(1 \pm r)^x$

Compound Interest

$A = P(1 + \frac{r}{n})^{nt}$

Continuously Compounded Interest

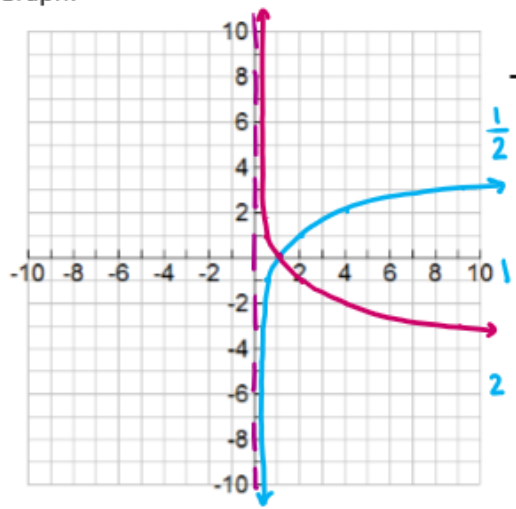
$A = Pe^{rt}$

Parent Function Name: Logarithmic

Equation: $y = \log_B x$

Ex: $y = \log_2 x$ Ex: $y = \log_{\frac{1}{2}} x$

Graph:



x	f(x)
$\frac{1}{2}$ 2 $\frac{1}{B}$	-1
1 1 1	0
2 $\frac{1}{2}$ B	1

Transformation Equation: $y = a \log_B\left(\frac{1}{b}(x-h)\right) + k$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

	growth	decay
End Behavior: As $x \to -\infty, f(x) \to$	DNE	DNE
As $x \to +\infty, f(x) \to$	∞	$-\infty$

x-intercept(s): (1, 0)

y-intercept: none

~~Vertex or~~ Critical Point: (1, 0)

Axis of Symmetry: none

Asymptotes: $x = 0$

Needed to Write Equation $y = \log_B(x-h) + k$

asymptote $x = h$

two points (x_1, y_1) and (x_2, y_2)
left right

Solve for k, then B

Additional Notes:

$$\log_B m^n = n \log_B m$$

$$\log_B mn = \log_B m + \log_B n$$

$$\log_B \frac{m}{n} = \log_B m - \log_B n$$

$$\log_b a = \frac{\log a}{\log b} = \frac{\ln a}{\ln b} = \frac{\log_c a}{\log_c b}$$

$$\log_B x = y \iff B^y = x$$